Homework 7

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A. ***Using resolution to prove that*** $(A \vee B) \wedge (\neg A \vee C) \wedge (\neg B \vee D) \wedge (\neg C \vee G) \wedge (\neg D \vee G) \vDash G$

Prove: *we can prove it by illustrating that* $(A \vee B) \wedge (\neg A \vee C) \wedge (\neg B \vee D) \wedge (\neg C \vee G) \wedge (\neg D \vee G) \wedge \neg G$ *is unsatisfiable. According to the* ***soundness of resolution****, we just need to prove the resolution closure of clauses in above sentence contains the empty clause.*

The resolving process is below:

$$\frac{\neg C \vee G, \neg G}{\neg C}, \quad \frac{\neg D \vee G, \neg G}{\neg D}, \quad \frac{\neg B \vee D, \neg D}{\neg B}, \quad \frac{A \vee B, \neg B}{A}, \quad \frac{\neg A \vee C, A}{\neg C}, \quad \frac{\neg C, C}{\phi}$$

Thus, $(A \vee B) \wedge (\neg A \vee C) \wedge (\neg B \vee D) \wedge (\neg C \vee G) \wedge (\neg D \vee G) \vdash G$.

Therefore, according to the **soundness of resolution**, we have $(A \vee B) \wedge (\neg A \vee C) \wedge (\neg B \vee D) \wedge (\neg C \vee G) \wedge (\neg D \vee G) \vDash G$.

B. ***How many semantically distinct 2-CNF clauses can be constructed from n proposition symbols?***

Solution: *n proposition symbols totally have* $2n$ *distinct literals. Therefore, the number of combinational cases from these literals is* $\binom{2}{2n} = 2n^2 - n$. *However, we don't consider about clauses like* $A \vee A$*, so the number need plus* $2n$. *Additional, all the clauses like* $\neg A \vee A$ *are logically equivalent, so we need subtract* $n-1$.

Therefore, the totally number of distinct 2-CNF clauses(denoted as N*) is:*

$$N = 2n^2 - n + 2n - (n-1) = 2n^2 + 1$$

*C.****Prove that propositional resolution always terminates in time polynomial in n given a 2-CNF sentence containing no more than n distinct symbols.***

Prove: *Trivially, two 2-CNF clauses resolving are always produce a 2-CNF clause. For instance,* $\frac{A \vee B, \neg A \vee C}{B \vee C}$. *According to the conclusion in* ***b****, the totally number of distinct 2-CNF clauses is* $(2n^2 + 1)$. *Then, the total number of possible distinct resolutions is* $(2n^2 + 1)^2$. *Also, any resolution step over two 2-CNF clauses is one of these* $(2n^2 + 1)^2$ *distinct resolutions. Therefore, this process terminates in time polynomial in n given a 2-CNF sentence containing no more than n distinct symbols.*

D. ***Explain why your argument in (c) does not apply to 3-CNF.***

Solution:

The algorithm above does not apply to 3-CNF because 3-CNF clauses are not closed under resolution operation. For example, $\frac{A \vee B \vee C, \neg A \vee D \vee F}{B \vee C \vee D \vee F}$. *Therefore, this property does not apply to 3-CNF clauses.*