# Homework 1

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## 1 Answer Sheet

**Theorem 1** *In the process of graph search, define the set of visited nodes as $V$, unexplored nodes as $U$ and the fringe as $F$. For any node $v \in V$, and any node $u \in U$, if there is a path from $v$ to $u$, the path must contain a node $f \in F$.*

**Proof 1** *Prove by contradiction. Assume that there is a path from $u$ to $v$ without any node in $F$. By the definition of fringe, for any node in $V$, its adjacent node is either in $V$, or in $F$. If there is no node in $F$ in the path from $v$ to $u$, then we can find at least one pair of nodes $s$ and $t$ in the path, where $s$ is in $V$ and $t$ in $U$, are adjacent. Thus $s$ has an adjacent node in neither $F$ nor $V$, which is a contradiction. So the assumption is wrong.*

**Theorem 2** *Uniform-cost search gives out the optimal solution, that is, the solution in which the sum of all the actions' cost is minimal.*

**Proof 2** *We claim that, for every node, the optimal path for it has been found when it is added to the explored set. This can be proved by contradiction and induction.*
*BS: The first node added into the explored set is the initial state, whose optimal solution as a goal state is obviously taking no action, costing zero.*
*IH: The optimal solutions for all the nodes in the explored set have been found.*
*IS: If the explored set is consisted of nodes whose optimal solution have been found and the set's fringe has also been constructed, then the lowest-cost node $f$ in the fringe has also found its optimal solution. Because if the real optimal path $p$ only consists of nodes in explored set and $f$, it is excatly this found solution, or at least as long as it. If not, the path must includes other nodes $f'$ in the fringe, $f$ can't be the lowest-cost node. So by contradiction, the optimal solution from initial state to $f$ has been found.*
*By induction, the claim above is ture. And since finally the goal state is added to the explored set, its optimal solution is found.*